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Conformal Invariance, Accelerating Universe and the Cosmological Constant Problem

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We investigate a conformal invariant gravitational model which is taken to hold at early universe. The conformal invariance allows us to make a dynamical distinction between the two unit systems (or conformal frames) usually used in cosmology and elementary particle physics. In this model we argue that when the universe suffers phase transition, the resulting mass scale introduced by particle physics should have a variable contribution to vacuum energy density. This variation is controlled by the conformal factor which is taken as a dynamical field. We then deal with the cosmological consequences of this model. In particular, we shall show that there is an inationary phase at early times. At late times, on the other hand, it provides a mechanism which makes a large effective cosmological constant relax to a sufficiently small value. Moreover, we shall show that the conformal factor acts as a quintessence field that leads the universe to accelerate at late times.

KEY WORDS: cosmology; cosmological constant.

1. INTRODUCTION

There is a fundamental conflict between observations and theoretical estimates on the value of the cosmological constant. In view of the cosmological observations we have an upper limit on the vacuum energy density which is equivalent to $\sim 10^{-29}$ g/cm^3. On the other hand, the standard model of particle physics implies that the universe has undergone a series of phase transitions at early epoch of its evolution contributing to the vacuum energy density 120 order of magnitude larger than this observational bound. Understanding of such a large discrepancy remains as one of the main problems of theoretical physics. There have been many attempts trying to resolve the problem (Weinberg, 1989). Most of them are based on the belief that the cosmological constant Λ may not have such an extremely small value at all the time and there should exist a dynamical mechanism working during evolution of the universe which provides a cancellation of the vacuum

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energy density at late times (Henneaux and Teitelboim, 1984; Abbott, 1985; Banks, 1985; Barr, 1987; Peccei *et al.*, 1987; Barr and Hochberg, 1988; Dolgov, 1983; Ford, 1987; Suen and Will, 1988).

Among different kinds of such models of a decaying cosmological constant, more promising ones may be those which consist of a scalar field nonminimally coupled to gravity which itself is described by the usual Einstein-Hilbert action with a cosmological term (Dolgov, 1983; Ford, 1987; Suen and Will, 1988). The scalar field evolves with cosmic expansion in such a way that its energy density which is unstable due to the gravitational interaction compensates the vacuum energy density. Generic feature of these models is that they result in an asymptotic behavior $\Lambda \sim t^{-2}$ which in the present epoch roughly gives the observed upper bound. Nevertheless, as an immediate consequence of a nonminimal coupling these models entail an effective gravitational coupling which also behaves asymptotically as $G_{\rm eff} \propto t^{-2}$, namely that gravity turns off at the cost of having a small cosmological constant.

This dramatic behavior encourages one to think about theories in which the gravitational coupling itself appears as a dynamical field, namely the scalar-tensor theories of gravitation. Along this line of thought we have concerned here with a particular form of these theories which is conformally invariant. The conformal invariance implies that the theory is invariant under local changes of units of length and time or local unit transformations (Dicke, 1962; Bekenstein and Meisels, 1980). In such transformations different unit systems or conformal frames are related via spacetime dependent conversion (or conformal) factors. The reason for introducing such a model to study the cosmological constant problem is the important fact that the observational estimates and the theoretical predictions are actually carried out in two different unit systems, the unit systems usually used in cosmology and elementary particle physics.[3] It is generally assumed that these two unit systems are related by a constant conversion factor. In other terms they are transformed by a global unit transformation.

In a local unit transformation, on the other hand, changes of unit systems find a dynamical meaning. We have already shown (Bisabr and Salehi, 2000) that these dynamical changes of unit systems can be taken as a basis for constructing a cancellation mechanism which reduces a large effective cosmological constant to a sufficiently small value. In the present work, we intend to study the effects of the model introduced in (Bisabr and Salehi, 2000) both on the early and the late times asymptotic behavior of the scale factor in the standard cosmological model. In particular, we wish to answer the questions that how this model affects the inflation at early times and whether it leads to an accelerating universe at late times.

[3] From now on these are referred to as the cosmological and the quantum frames(unit systems), respectively.

We shall assume that gravity is described by a conformal invariant gravitational model (action (1)) at early universe, speciffcally before breaking the gauge symmetry of fundamental interactions at GUT energy scale. When the gauge symmetry is spontaneously broken the resulting vacuum energy density as a constant mass scale introduced by particle physics in the quantum frame, breaks down the conformal invariance. This mass scale may be considered in different conformal frames by local changes of the unit system. It means that the vacuum energy density takes a variable configuration in the cosmological frame. This variation is controlled by the conformal factor that appears as a dynamical field in our model. We shall show that this dynamical field plays a key role in a cancellation mechanism that works essentially due to expansion of the universe. It also appears as a quintessence field which causes the universe to accelerate at late times.

We organize this paper as follows: In Section 2, we introduce a conformal invariant gravitational theory consisting of a real scalar field which is conformally coupled to gravity. It is the classical analogue of the model which we have already investigated in (Salehi *et al.*, 2000; Salehi and Bisabr, 2000). In Section 3, it is argued that incorporation of an effective cosmological term to the model requires that one makes a dynamical distinction between the cosmological and the quantum unit systems. The dynamical distinction means that the two unit systems are related by a spacetime dependent conversion factor. We then study the consequences of such a distinction in two parts. Firstly, investigation of the model at early times reveals that it can bring the universe into an inationary phase. Secondly, we shall show that while the model leads to a damping behavior for the effective value of the cosmological constant at late times, it avoids the aforementioned problem on the gravitational coupling. Moreover, evolution of the scale factor indicates that the model gives rise to an accelerating expansion for the universe at late times. In Section 4, we summarize and discuss our results.

Throughout this paper we work in units in which $\hbar = c = 1$ and the sign conventions are those of MTW (Misner *et al.*, 1973).

2. THE MODEL

We use a gravitational system which consists of a real scalar field ϕ and the gravitational field, described by the action[4]

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{6}R\phi^2\right) \qquad (1)$$

where ∇_μ denotes a covariant differentiation and R is the Ricci scalar. Note that the action does not involve the free gravitational field contribution.

[4] This action has been investigated in different contexts. See, for example, (Bekenstein and Meisels, 1980; Deser, 1970)

The remarkable feature of (1) is that it is invariant under conformal transformations

$$g_{\mu\nu} = e^{2\sigma}\bar{g}_{\mu\nu} \tag{2}$$

$$\phi(x) = e^{-\sigma}\bar{\phi}(x) \tag{3}$$

where σ is a smooth dimensionless spacetime function. This means that the theory described by the action (1) can be considered in many different conformal frames which are dynamically equivalent. They correspond to various configurations one assigns to the scalar field ϕ or various choices of local standards of units. Therefore different conformal frames may be distinguished by local values of some dimensional parameters which enter the theory.

Variation of (1) with respect to $g^{\mu\nu}$ and ϕ yields, respectively,

$$G_{\mu\nu} = -6\phi^{-2}\left(\nabla_\mu\phi\nabla_\nu\phi - \tfrac{1}{2}g_{\mu\nu}\nabla_\gamma\phi\nabla^\gamma\phi\right) + \phi^{-2}(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\phi^2 \tag{4}$$

and

$$\Box\phi - \frac{1}{6}R\phi = 0 \tag{5}$$

Here $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$ and $G_{\mu\nu}$ is the Einstein tensor. One should recognize that Eqs. (4) and (5) are not independent. Indeed, the trace of (4) gives

$$\phi\left(\Box - \frac{1}{6}R\right)\phi = 0 \tag{6}$$

which contains Eq. (5). This is a direct consequence of the absence of a dimensional parameter in the model. In the next section we shall introduce a cosmological term which leads the field equations to be independent.

3. COSMOLOGICAL IMPLICATIONS

3.1. Vacuum-Dominated Era

It is generally believed that at GUT energy scale the universe has passed through a certain disordered phase associated with gauge symmetry of the grand unified theories. When the gauge symmetry is spontaneously broken the structure of the vacuum drastically changes in the sense that it acquires a large amount of energy density which appears as a large effective cosmological constant. The key question which should be answered at this stage is that how this vacuum energy density should be coupled to gravity.

To clarify this point we remark that the use of two different unit systems are conventional for measuring this energy density. On one hand, the upper bound set by observations is obtained in a unit system which is defined in terms of large-scale cosmological parameters (the cosmological unit system). On the other hand, the theoretical predictions are based on a natural unit system which is suggested by quantum physics (the quantum unit system). One faces with a large discrepancy between observations and theoretical predictions on vacuum energy density if one presupposes that these two unit systems are indistinguishable up to a constant conversion factor in all spacetime points. It means that they transform to each other by a global unit transformation. Such a global transformation clearly carries no dynamical implications and the use of a particular unit system is actually a matter of convenience.

The point we wish to make here is that for introducing a dynamical mechanism to reduce a large vacuum energy density one needs to note that such a mechanism should work whenever the contributions of quantum physics to this vacuum density are considered at cosmological level since the cosmological constant problem arises when one compares these contributions with relevant cosmological observations. This point strongly suggests that construction of a mechanism for relaxing these contributions should somehow take into account the distinction between the cosmological and the quantum unit systems. In order to establish such a distinction we would like to consider local unit transformations. We introduce a theoretical scheme in which an explicit recognition is given to the distinguished characteristics of the cosmological and the quantum frames. In such a theoretical scheme one should no longer accept the triviality one usually assigns to a unit transformation.

We first assume that gravity is described by the conformal invariant gravitational model (1). We also assume that the cosmological and the quantum frames are described by $g^{\mu\nu}$, ϕ and $\bar{g}^{\mu\nu}$, $\bar{\phi}$ respectively. When the universe goes through phase transition, the vacuum energy density takes a nonzero expression that appears in the action (1) as a large mass scale. This mass scale has a preferred constant configuration in the quantum frame, denoted in the following by $\bar{\Lambda}$, which is theoretically suggested by elementary particle physics. In this case there exists a constant scale of length that incorporates a distinction between the standard of units, and therefore leads to breakdown of the conformal invariance of (1). It should be noted that this distinction is a dynamical distinction in the sense that two different unit systems are linked by a local unit transformation.

To incorporate $\bar{\Lambda}$ into the action (1), we write it in the cosmological frame by a local change of the unit system, namely

$$\Lambda = \bar{\Lambda} e^{-2\sigma} \tag{7}$$

In the relation (7), Λ and $\bar{\Lambda}$ denote the cosmological constants in the cosmological and the quantum frames, respectively. Note that since $\bar{\Lambda}$ carries the dimension of squared mass, Λ takes an exponential factor $e^{-2\sigma}$. Thus Λ has not actually a constant configuration in the cosmological frame.

Action (1) then takes the form

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\{g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \tfrac{1}{6}(R - 2\bar{\Lambda}e^{-2\sigma})\phi^2\} \tag{8}$$

To study the evolution of the cosmological term Λ, we let the above action involve a kinetic term for σ. In this way we consider σ as a dynamical field. This seems to be necessary to account for the dynamical distinction between the cosmological and the quantum unit systems as it is implied by the relation (7). We therefore write the action (8) as

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\left\{g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \left(\frac{1}{6}(R - 2\bar{\Lambda}e^{-2\sigma}) + \alpha g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma\right)\phi^2\right\} \tag{9}$$

where α is a dimensionless constant parameter. This action can now be used to describe a vacuum-dominated universe since it excludes any matter contribution.

Variation of (9) with respect to $g^{\mu\nu}$, ϕ and σ yields, respectively,

$$G_{\mu\nu} + \bar{\Lambda}e^{-2\sigma}g_{\mu\nu} = 6\phi^{-2}\tau_{\mu\nu} \tag{10}$$

$$\Box\phi - \frac{1}{6}R\phi + \frac{1}{3}\bar{\Lambda}\phi e^{-2\sigma} - \alpha\phi\nabla_\gamma\sigma\nabla^\gamma\sigma = 0 \tag{11}$$

$$\frac{1}{\sqrt{-g}}\nabla_\mu\left(\sqrt{-g}\phi^2 g^{\mu\nu}\nabla_\nu\sigma\right) = \frac{1}{3\alpha}\bar{\Lambda}\phi^2 e^{-2\sigma} \tag{12}$$

where

$$\begin{aligned}\tau_{\mu\nu} = {} & -\left(\nabla_\mu\phi\nabla_\nu\phi - \tfrac{1}{2}g_{\mu\nu}\nabla_\gamma\phi\nabla^\gamma\phi\right) + \frac{1}{6}(\nabla_\mu\nabla_\nu - g_{\mu\nu})\phi^2 \\ & -\alpha\phi^2\left(\nabla_\mu\sigma\nabla_\nu\sigma - \tfrac{1}{2}g_{\mu\nu}\nabla_\gamma\sigma\nabla^\gamma\sigma\right)\end{aligned} \tag{13}$$

The exponential coefficient of $\bar{\Lambda}$ emphasizes that this mass scale belongs to a unit system which is different from that used in cosmology.

Intuitively, one expects that there should be no distinction between the cosmological and the quantum unit systems at sufficiently early times so that

$$\begin{aligned} e^{-2\sigma} &\to 1 \\ \Lambda = \bar{\Lambda}e^{-2\sigma} &\to \bar{\Lambda} \quad \text{as} \quad t \to 0 \end{aligned} \tag{14}$$

This can be taken as an early-time boundary condition for the dynamical field σ. In an expanding universe the distinction between these two unit systems is expected to increase with time since all cosmological scales enlarge as the universe expands. Thus $e^{2\sigma}$ must be an increasing function of time. According to (7), this automatically provides us with a dynamical mechanism for reducing the mass scale Λ in the cosmological frame.

Before studying this mechanism we would like to focus on the behavior of the field equations at early times. To do this, we apply the field equations to a homogeneous and isotropic universe. In particular, we specialize to a spatially flat Friedman-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{15}$$

where $a(t)$ is the scale factor. The homogeneity and isotropy require that the fields ϕ and σ be only functions of time. The Eqs. (10)–(12) become

$$3\left(\frac{\dot{a}}{a}\right)^2 - \bar{\Lambda}e^{-2\sigma} + 3\frac{\dot{\phi}^2}{\phi^2} + 6\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + 3\alpha\dot{\sigma}^2 = 0 \tag{16}$$

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \alpha\dot{\sigma}^2 - \frac{1}{3}\bar{\Lambda}e^{-2\sigma} = 0 \tag{17}$$

$$\ddot{\sigma} + \left(3\frac{\dot{a}}{a} + 2\frac{\dot{\phi}}{\phi}\right)\dot{\sigma} + \frac{\bar{\Lambda}}{3\alpha}e^{-2\sigma} = 0 \tag{18}$$

where the overdot indicates differentiation with respect to t. We take

$$a \sim e^{nt} \tag{19}$$

$$\phi \sim e^{mt} \tag{20}$$

and

$$\sigma = \xi t \tag{21}$$

We substitute these and the condition (14) into (16), (17) and (18) to obtain

$$n = \xi = \frac{1}{\sqrt{\alpha(4\alpha + 1)}}\sqrt{\frac{1}{3}\Lambda} \tag{22}$$

$$m = (2\alpha - 1)n \tag{23}$$

These results indicate that the scale factor grows exponentially and the spacetime geometry is described by the de Sitter metric. This inflation continues to be a solution until $e^{-2\alpha} \approx 1$ holds. After a sufficiently long time this approximation is no longer valid and the ination ends.

Therefore, this model does not provide any contradiction with existence of an inflationary phase at early times. This is important since the basic idea of inflation seems to be the only reasonable program, suggested so far, to resolve the cosmological puzzles such as the flatness and the horizon problems (Guth, 1981; Linde, 1982; Albrecht and Steinhardt, 1982; La and Steinhardt, 1989).

3.2. Matter-Dominated Era

To apply the model to a matter-dominated universe we should first introduce a matter system in action (9). We write

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}\Bigg\{g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \phi^2\left(\frac{1}{6}(R - 2\bar{\Lambda}e^{-2\sigma}) + \alpha g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma\right)\Bigg\} + S_m[g_{\mu\nu}] \tag{24}$$

where $S_m[g_{\mu\nu}]$ is the matter field action. The gravitational equation for action (24) will be

$$G_{\mu\nu} + \bar{\Lambda}e^{-2\sigma}g_{\mu\nu} = 6\phi^{-2}(T_{\mu\nu} + \tau_{\mu\nu}) \tag{25}$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}S_m[g_{\mu\nu}] \tag{26}$$

The field equations of ϕ and σ remain unchanged in the presence of matter. We may put the Eq. (11) into the trace of (25) to obtain

$$T^\gamma_\gamma = \frac{1}{3}\bar{\Lambda}\phi^2 e^{-2\sigma} \tag{27}$$

which is a relation between the trace of the matter stress-tensor and the vacuum energy density. This relation has important implications that we describe in the following:

Let us first take $T_{\mu\nu}$ to be the stress-tensor of a perfect fluid with energy density ρ and pressure p, namely

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu) \tag{28}$$

where u_μ is the four velocity of the fluid. For a pressureless fluid, (27) takes the form

$$\rho\phi^{-2} \sim \bar{\Lambda}e^{-2\sigma} \tag{29}$$

In an expanding universe $\rho\phi^{-2}$ decreases. The relation (29) predicts that the same thing happens for Λ. Two important results arise from this statement. Firstly, the expansion of the universe induces the reduction of Λ. This requires that the conformal factor $e^{2\sigma}$, or the σ field, be an increasing function of time. We previously discussed that this is indeed the case in a vacuum-dominated universe. We shall show that this is also true in a matter-dominated universe.

Secondly, the dynamical distinction which we have made between the cosmological and the quantum frames does not allow to incorporate naively a constant mass scale such as $\bar{\Lambda}$ into the action (24). Due to (27), this would not be dynamically consistent with the field equations. We intend now to investigate the field equations in a matter-dominated universe. For the metric (15), the Eq. (25) becomes

$$3\left(\frac{\dot{a}}{a}\right)^2 - \bar{\Lambda}e^{-2\sigma} + 3\frac{\dot{\phi}^2}{\phi^2} + 6\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + 3\alpha\dot{\sigma}^2 = 6\phi^{-2}\rho \tag{30}$$

For late times we take

$$a \sim \left(\frac{t}{t_0}\right)^v \tag{31}$$

$$\phi = \text{const.} \tag{32}$$

$$e^\sigma = \sigma_0 t \tag{33}$$

where t_0 is the present age of the universe and σ_0 is a constant with dimension of mass. Let us first estimate the gravitational coupling. To do this, we substitute (32) into Eq. (30) to obtain

$$3H^2 - \bar{\Lambda}e^{-2\sigma} + 3\alpha\dot{\sigma}^2 = 6\phi^{-2}\rho \tag{34}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Using (29) this equation becomes

$$3H^2 + 3\alpha\dot{\sigma}^2 \sim \phi^{-2}\rho \tag{35}$$

From the relation (33), one infers that $\dot{\sigma} \to t^{-1} \sim H$. Thus (35) reduces to

$$H^2 \sim \phi^{-2}\rho \tag{36}$$

Now we may use the observational fact that (Weinberg, 1972)

$$\rho \sim \rho_c \tag{37}$$

with ρ_c being the critical density

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{38}$$

Here G is the gravitational constant and H_0 is the Hubble constant. When $t \to t_0$, we obtain from (36) and (37)

$$\phi^{-2} \sim G \tag{39}$$

Thus the constant configuration of ϕ^{-2} at late times is given by the gravitational constant. In this case the action (24) reduces to

$$S = \frac{-1}{16\pi G}\int d^4x\sqrt{-g}(R - 2\bar{\Lambda}e^{-2\sigma} + 6\alpha g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma) + S_m[g_{\mu\nu}] \tag{40}$$

This differs from the usual Einstein-Hilbert action in the sense that it contains a dynamical field σ and a varying cosmological term $\bar{\Lambda}e^{-2\sigma}$.

Variation of (40) with respect to $g^{\mu\nu}$ and σ gives the field equations

$$G_{\mu\nu} + \bar{\Lambda} e^{-2\sigma} + 6\alpha \left(\nabla_\mu \sigma \nabla_\nu \sigma - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \sigma \nabla^\gamma \sigma \right) = 8\pi G T_{\mu\nu} \tag{41}$$

$$\Box \sigma = \frac{\bar{\Lambda}}{3\alpha} e^{-2\sigma} \tag{42}$$

For the metric (15) and the matter stress-tensor (28), these equations become

$$3\frac{\dot{a}^2}{a^2} - \bar{\Lambda} e^{-2\sigma} + 3\alpha\dot{\sigma} = 8\pi G\rho \tag{43}$$

$$\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} - \bar{\Lambda} e^{-2\sigma} - 3\alpha\dot{\sigma} = -8\pi G p \tag{44}$$

$$\ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} + \frac{\bar{\Lambda}}{3\alpha} e^{-2\sigma} = 0 \tag{45}$$

Eq. (43) together with (44) gives

$$3\frac{\ddot{a}}{a} - \bar{\Lambda} e^{-2\sigma} - 6\alpha\dot{\sigma}^2 = -4\pi G(\rho + 3p) \tag{46}$$

For $p = 0$, we combine this with (43) to obtain

$$\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} - 3\alpha\dot{\sigma}^2 - \bar{\Lambda} e^{-2\sigma} = 0 \tag{47}$$

Now if we substitute (31) and (33) in the equations (45) and (47) we obtain

$$v = \tfrac{2}{3}, \quad -3\alpha \tag{48}$$

$$\sigma_0 = \sqrt{\frac{\bar{\Lambda}}{3\alpha(1 - 3v)}} \tag{49}$$

One can take the solution $v = \frac{2}{3}$ for $\alpha < 0$ since only in this case $\sigma_0^2 > 0$. This corresponds to the solution of the standard cosmological model for the evolution of the scale factor in a matter-dominated universe. For $v = -3\alpha$, we obtain accelerating solutions (with $v > 1$) for $\alpha < -\frac{1}{3}$. The deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \tag{50}$$

is

$$q = \frac{1}{v} - 1 \tag{51}$$

which is negative for $\alpha < -\frac{1}{3}$.

On the other hand, if we use (33) and (49) we obtain for the cosmological term

$$\Lambda = \bar{\Lambda}^{-2\sigma} \sim t^{-2} \tag{52}$$

which is consistent with the observational bound.

We see that the conformal factor (or the σ field) plays two important roles in our model. Firstly, evolution of this dynamical field induced by the cosmic expansion damps a large effective cosmological constant. Secondly, it plays the role of a quintessence field that causes the universe to accelerate at late times.

4. SUMMARY AND DISCUSSION

We have investigated the cosmological consequences of a conformal invariant gravitational model which is assumed to hold during the very early stages of evolution of the universe. The conformal invariance of the model allows us to formalize a theoretical framework in which there exists a dynamical distinction between the two unit systems used in cosmology and elementary particle physics. It is argued that when the universe goes through phase transition the resulting large effective cosmological constant $\bar{\Lambda}$, as a mass scale introduced by particle physics, is related to the corresponding mass scale in the cosmological unit system by $\Lambda = \bar{\Lambda} e^{-2\sigma}$. We have shown that this results in a damping behavior for Λ caused by expansion of the universe. We emphasize that this feature is also suggested by the relation (27) which is a dynamical consistency relation on the trace of the matter stress-tensor in our gravitational system.

The question which naturally arises is whether such a variable cosmological term alters the standard picture of early history of the universe. To address this question, we have shown that there exists a solution for the field equations at early times exhibiting an exponential growth of the scale factor. It is important to note that in this model there is a natural exit of the universe from this inflationary phase, namely when $e^{2\sigma} \approx 1$ does not hold due to growth of σ.

Our primary interest is to explore the cosmological constant problem. We have shown that the asymptotic solution of the field equations in the matter-dominated era leads to the following consequences:

1. The relation (52) indicates that the cosmological constant in the cosmological frame Λ is of the same order of t^{-2} which is consistent with the upper bound set by observations. The smallness of the cosmological constant is therefore related to the fact that the universe is old.
2. The gravitational coupling in the present state of the universe is given by $\phi^{-2}\,\sigma G$.
3. The scale factor exhibits a late-time asymptotic power law expansion $a \propto t^{\nu}$ with $\nu > 1$.

This implies that the universe is accelerating and σ plays the role of a quintessence field. The acceleration of the universe is generally achieved by negative values of α ($\alpha < -\frac{1}{3}$). This means that σ is a massless scalar field with positive energy density.

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